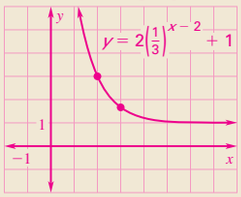
Extra Example 3

Graph $y = 2\left(\frac{1}{3}\right)^{x-2} + 1$. State the domain and range. **domain: all real numbers; range: $y > 1$**



Extra Example 4

A new car costs $25,000. The value of the car decreases by 15% each year.

- Write an exponential decay model giving the car's value y (in dollars) after t years. Estimate the value after 4 years. **$y = 25{,}000(0.85)^t$; about $13,050**
- Graph the model.

- Use the graph to estimate when the value of the car will be $8000. **after about 7 yr**

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: What does the graph of an exponential decay function look like?

- **An exponential decay function is a function of the form $y = ab^x$ with $a > 0$ and $0 < b < 1$.**
- **Exponential decay models describe situations in which a quantity decreases by a fixed percent each time period.**

The graph of an exponential decay function is a curve that falls from left to right and gets less and less steep as x increases. The x-axis or a line parallel to it is a horizontal asymptote of the graph.

EXPONENTIAL DECAY MODELS When a real-life quantity decreases by a fixed percent each year (or other time period), the amount y of the quantity after t years can be modeled by the equation

$$y = a(1 - r)^t$$

where a is the initial amount and r is the percent decrease expressed as a decimal. Note that the quantity $1 - r$ is the decay factor.

EXAMPLE 4 Solve a multi-step problem

SNOWMOBILES A new snowmobile costs $4200. The value of the snowmobile decreases by 10% each year.

- Write an exponential decay model giving the snowmobile's value y (in dollars) after t years. Estimate the value after 3 years.
- Graph the model.
- Use the graph to estimate when the value of the snowmobile will be $2500.

Solution

STEP 1 The initial amount is $a = 4200$ and the precent decrease is $r = 0.10$. So, the exponential decay model is:

$y = a(1 - r)^t$ — Write exponential decay model.

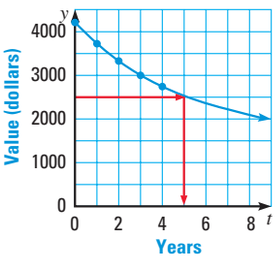
$= 4200(1 - 0.10)^t$ — Substitute 4200 for a and 0.10 for r.

$= 4200(0.90)^t$ — Simplify.

When $t = 3$, the snowmobile's value is $y = 4200(0.90)^3 = \$3061.80$.

AVOID ERRORS Notice that the percent decrease, 10%, tells you how much value the snowmobile *loses* each year. The decay factor, 0.90, tells you what fraction of the snowmobile's value *remains* each year.

STEP 2 The graph passes through the points (0, 4200) and (1, 3780). It has the t-axis as an asymptote. Plot a few other points. Then draw a smooth curve through the points.



STEP 3 Using the graph, you can estimate that the value of the snowmobile will be $2500 after about 5 years.

GUIDED PRACTICE for Examples 3 and 4

Graph the function. State the domain and range. **4–6. See margin.**

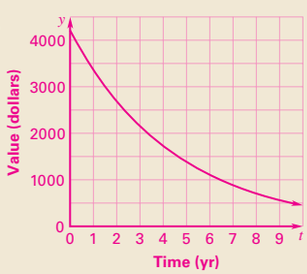
4. $y = \left(\frac{1}{4}\right)^{x-1} + 1$ **5.** $y = 5\left(\frac{2}{3}\right)^{x+1} - 2$ **6.** $g(x) = -3\left(\frac{3}{4}\right)^{x-5} + 4$

7. WHAT IF? In Example 4, suppose the value of the snowmobile decreases by 20% each year. Write and graph an equation to model this situation. Use the graph to estimate when the value of the snowmobile will be $2500. **$y = 4200(0.8)^t$, see margin for art; after about 2 yr.**

8. SNOWMOBILE The value of a snowmobile has been decreasing by 7% each year since it was new. After 3 years, the value is $3000. Find the original cost of the snowmobile. **$3729.69**

4–6. See Additional Answers beginning on p. AA1.

7.



7.2 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 9, 19, and 33

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 15, 27, 28, 33, and 35

SKILL PRACTICE

A

1. VOCABULARY In the exponential decay model $y = 1250(0.85)^t$, identify the initial amount, the decay factor, and the percent decrease. **1250, 0.85, 15%**

2. ★ WRITING *Explain* how to tell whether the function $y = b^x$ represents exponential growth or exponential decay. **If b is greater than 1, then the function represents exponential growth. If b is greater than 0 and less than 1, the function represents exponential decay.**

CLASSIFYING FUNCTIONS **Tell whether the function represents *exponential growth* or *exponential decay*.**

3. $f(x) = 3\left(\frac{3}{4}\right)^x$ **exponential decay**

4. $f(x) = 4\left(\frac{5}{2}\right)^x$ **exponential growth**

5. $f(x) = \frac{2}{7} \cdot 4^x$ **exponential growth**

6. $f(x) = 25(0.25)^x$ **exponential decay**

EXAMPLES 1 and 2 on p. 486–487 for Exs. 7–15

GRAPHING FUNCTIONS **Graph the function.** **7–14. See margin.**

7. $y = \left(\frac{1}{4}\right)^x$

8. $y = \left(\frac{1}{3}\right)^x$

9. $f(x) = 2\left(\frac{1}{5}\right)^x$

10. $y = -(0.2)^x$

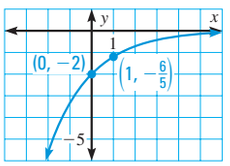
11. $y = -4\left(\frac{1}{3}\right)^x$

12. $g(x) = 2(0.75)^x$

13. $y = \left(\frac{3}{5}\right)^x$

14. $h(x) = -3\left(\frac{3}{8}\right)^x$

15. ★ MULTIPLE CHOICE The graph of which function is shown? **B**



(A) $y = 2\left(-\frac{3}{5}\right)^x$ **(B)** $y = -2\left(\frac{3}{5}\right)^x$

(C) $y = -2\left(\frac{2}{5}\right)^x$ **(D)** $y = 2\left(-\frac{2}{5}\right)^x$

EXAMPLE 3 **B** on p. 487 for Exs. 16–25

TRANSLATING GRAPHS **Graph the function. State the domain and range.** **16–24. See margin.**

16. $y = \left(\frac{1}{3}\right)^x + 1$

17. $y = -\left(\frac{1}{2}\right)^{x-1}$

18. $y = 2\left(\frac{1}{3}\right)^{x+1} - 3$

19. $y = \left(\frac{2}{3}\right)^{x-4} - 1$

20. $y = 3(0.25)^x + 3$

21. $y = \left(\frac{1}{3}\right)^{x-2} + 2$

22. $f(x) = -3\left(\frac{1}{4}\right)^{x-1}$

23. $g(x) = 6\left(\frac{1}{2}\right)^{x+5} - 2$

24. $h(x) = 4\left(\frac{1}{2}\right)^{x+1}$

25. GRAPHING CALCULATOR Consider the exponential decay function $y = ab^{x-h} + k$ where $a = 3$, $b = 0.4$, $h = 2$, and $k = -1$. Predict the effect on the function's graph of each change in a, b, h, or k described in parts (a)–(d). Use a graphing calculator to check your prediction.

a. a changes to 4

b. b changes to 0.2

c. h changes to 5

d. k changes to 3

25a. The graph is a vertical stretch by a factor of $\frac{4}{3}$.

25b. The graph will be steeper because the decay factor is smaller.

25c. The graph moves 5 units to the right instead of 2 units to the right.

25d. The horizontal asymptote moves to $x = 3$.

26. ERROR ANALYSIS You invest \$500 in the stock of a company. The value of the stock decreases 2% each year. *Describe* and correct the error in writing a model for the value of the stock after t years.

$y = (\text{Initial amount})(\text{Decay factor})^t$

$y = 500(0.02)^t$ ✗

The decay factor is $1 - r$, not r; $y = 500(0.98)^t$.

4 PRACTICE AND APPLY

Assignment Guide

Answer Transparencies available for all exercises

Basic:
Day 1: pp. 489–491
Exs. 1–10, 15–19, 30–34, 37–49 odd

Average:
Day 1: pp. 489–491
Exs. 1–6, 9–12, 15, 19–27, 30–35, 38, 41, 44, 47, 49

Advanced:
Day 1: pp. 489–491
Exs. 1, 2, 10–15, 20–36*, 39, 42, 45, 47, 49

Block:
pp. 489–491
Exs. 1–6, 9–12, 15, 19–27, 30–35, 38, 41, 44, 47, 49 (with 7.1)

Differentiated Instruction

See *Algebra 2 Best Practices Toolkit* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 8, 10, 16, 18, 30

Average: 9, 12, 20, 22, 30

Advanced: 10, 14, 21, 24, 31

Extra Practice

- Student Edition, p. 1016
- Chapter 7 Resource Book: Practice levels A, B, C, pp. 19–21

Practice Worksheet

An easily-readable reduced practice page (with answers) for this lesson can be found on p. 476C.

7–24. See Additional Answers beginning on p. AA1.